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Why is It so Hard to Solve Long Divisions for 10-Year-Old Children?

Camos V* **and Baumer J**

Université de Fribourg, Fribourg Center for Cognition, Département de Psychologie, Rue de Faucigny 2, 1700 Fribourg, Switzerland

***Corresponding author:** Valérie Camos, Université de Fribourg, Fribourg Center for Cognition, Département de Psychologie, Ruede Faucigny 2, 1700 Fribourg, Switzerland, Tel: +41 26 300 7675; Fax: +41 26 300 9712; E-mail: valerie.camos@unifr.ch

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Abstract

Although division is considered as the most difficult arithmetic operation to solve, it is also the less studied. The aim of the present study was to explore the contribution of some cognitive abilities (knowledge of multiplicative facts, attentional and spatial capacities) of 10-year-old children to their achievement in solving long divisions, and to define the factors that make long divisions difficult. Although the size of the operands predicted performance, the number of processing steps required to perform each division was the best predictor of achievement. Moreover, increased attention capacity and better knowledge in multiplicative facts favoured division solving, whereas spatial capacity did not contribute for unique variance. Finally, the decrease in performance when solution requires more processing steps was stronger in children with lower attentional capacity.

Keywords: Division; Arithmetic; Attention; Procedure

Introduction

In contemporary society, knowledge in mathematics is crucial to succeed in education and professional life. This relationship is even stronger nowadays than some decades ago because an increased number of jobs require mathematical proficiency [1]. Recently, a large cross-national study found that primary school pupils' knowledge of fraction and division at age 10 is a unique predictor of their attainment in algebra and overall mathematics performance five or six years later [2]. Although division is important for future achievement in education, and often considered as the most difficult arithmetic operation, it is less studied than the other arithmetic operations. While some studies focused on the conceptual understanding of division in younger children [3-6], few studies were dedicated to simple division, and even less to the solving of Euclidian or long division. The aim of the present study was to explore some cognitive abilities of 10 year old children that may contribute to their achievement in solving long divisions, and to define the factors that make their solving difficult. Contrasting with studies on other arithmetic operations, very few studies were dedicated to Euclidian or long division [7-10]. Moreover, these studies focus on variations in strategies used by Dutch children to solve long division, because in Dutch primary schools there is no formal teaching of an algorithm, contrary to what is done in France. Euclidean division is a conventional algorithm of division of two integers producing two other integers. In the process, the dividend is divided by the divisor to produce a result called the quotient and a remainder (i.e., dividend/divisor = quotient + remainder). The algorithm breaks the division into a series of elementary steps that can be performed by hand. The process begins by dividing the leftmost digit of the dividend by the divisor. The quotient (rounded down to an integer) becomes the leftmost digit of the result, and the remainder is calculated (this step is notated as a subtraction).

This remainder carries forward when the process is repeated on the following digit of the dividend (notated as 'bringing down' the next digit to the remainder). When all digits of the dividend have been processed, the process is completed. Contrary to simple division, long division always involves several computational steps, and cannot be solved by a single retrieval from long-term memory. Even the simplest long division of one-digit number divided by one-digit number (e.g., 5/2) requires to compute a quotient and a remainder. Thus, in such simple cases, a long division would involve at the minimum two retrievals, for a division fact (4/2=2) and a subtraction fact (5-4=1). Across countries, the computation algorithm is quite similar, but is written down differently (see an example of the algorithm used in France in Figure 1) Like the other arithmetic operations, divisions could be segregated into simple and complex. Simple divisions refer to operations involving two one-digit numbers, the other divisions being complex. To have a more fine grained classification of long divisions, we took the classification proposed by Boucheny and Guérinet [11] into four types, which takes into account the number of digits in the divisor and in the quotient. Type 1 divisions involve one-digit numbers in both divisor and quotient (e.g., 56/9), while Type 2 divisions involve one-digit number for the divisor, but multi-digit number in the quotient (e.g., 94/7). Type 3 divisions have multi-digit number as divisor but lead to a one-digit quotient (e.g., 99/26), and Type 4 divisions involve multi-digit numbers both in divisor and quotient (e.g., 768/36). Thus, this classification proposes a progression in the complexity of the divisions, Type 1 being the simplest as it could be solved through retrievals whereas Type 4 requires numerous computational steps.

The aim of our study was to examine some of the skills or capacities that would affect children's performance in long division. In his review about the core deficits that distinguish children with mathematical disabilities from their peers, Geary [12] identified three main dimensions: (1) the representation and retrieval of mathematical facts from long-term memory, (2) the computational algorithms and (3) the visuospatial skills. These three dimensions are fundamental for mathematics achievement, and we believe that they would also affect the solving of long division. Concerning the first dimension, studies on simple divisions lead to expect that knowledge in multiplication facts would be a determinant factor to solve long division. These studies in young adults focused on how multiplication and division facts are related in memory. While some authors suggest that multiplication (e.g., 6 x 8=48) and division (e.g., 48/6=8) facts are stored and retrieved as independent representations [13,14], others proposed that multiplication could be used as a check after direct retrieval of the quotient [15-17]. For children, the results of the sole longitudinal study in 8 year old children were more in line with the idea of a shared memory network for multiplication and division facts [18], and in accordance with this idea, Robinson et al. [19] observed that multiplication is the predominant strategy to solve simple division between Grade 5 and 7. The impact of multiplicative facts knowledge would be magnified in long division, because most of the long divisions need multiple and successive retrievals. A quick and reliable access to multiplicative facts would reduce reliance on effortful strategies like recursive additions to compute a product, and thus reduce the number of processing steps and the possibilities of committing errors. In the present study, we thus evaluated knowledge in multiplication facts of 10-year-old children by asking them to answer a maximum of simple multiplications among 80 multiplications presented for two minutes. We chose to assess only multiplication facts in accordance with the fact that multiplication and division facts share the same mnemonic network in children of that age, and because division facts are not formally taught in France, contrary to multiplication facts.

Concerning the computational procedure, the learning of the algorithm for long division is out of the focus of the present study. Although this is probably the most difficult algorithm for solving arithmetic problems and an important educational challenge, our aim was to identify the cognitive abilities that would affect the solving of long division in children who had previously learned the algorithm. To control for the type of algorithm used to solve long division and for schooling effect, we chose to test pupils attending French primary schools in which there is a national curriculum defining learning objectives and teaching methods. As a consequence, all pupils use the same algorithm, and the 10-year-old children involved in the present study all started to learn the algorithm for long division a year before. Although this study did not focus on the learning of the algorithm, we were interested in the impact of the complexity of the computational algorithm. Because divisions differ from each other on the number of processing steps they require, we expected that long divisions with more computational steps should lead to poorer performance, even in children who learnt the algorithm. Finally, concerning the last dimension identified by Geary [12], previous research has established a relation between spatial skills and achievement in mathematics across a variety of spatial tasks [20-23] and mathematical domains [7]. The fact that spatial skills predict performance on broad measures of math achievement such as the SAT-M or word problems [21,22,24,25] suggests that spatial processing might be implicated in other domains of mathematics beyond those that are ostensibly spatial, like geometry.

More specifically, Trbovich and LeFevre [26] showed that mental addition requires more visual resources when presented vertically instead of horizontally. Long divisions are necessarily presented vertically, and solving such problems demands a neat spatial organization of the digits (Figure 1). Divisions requiring to write down several remainders and to do several "bringing-downs" lead to more opportunities of errors. They should be less correctly solved due to their higher spatial load. Moreover, children with poorer visuospatial skills should experience more difficulties in the spatial organization of the digits. We thus evaluated children's capacity in visuospatial planning with the Mazes subtest from the WISC III [27] and the Rover subtest from the Kaufman Assessment Battery for Children [28].

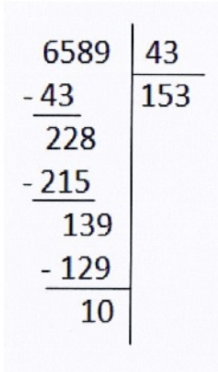

Figure 1: An example of the canonical algorithm used in France to solve Euclidian divisions.

Finally, we chose to assess capacity in selective attention with two tests of visual selective attention that can be administered in groups: the attention and concentration test d2 [29] and the Cancellation subtest from WISC-IV [26]. Selective attention refers to the processes that select and focus on particular input relevant for further processing while inhibiting irrelevant or distracting information. Besides the three main components identified by Geary [12], we think that individual differences in selective attention should also affect the performance in solving long divisions (cf. for review on the role of selective attention in school achievements, [30].

It was already showed that children with mathematical difficulties performed worse than controls in a selective attention measure [31]. Solving long divisions should depend on selective attention in children because this requires multiple processing steps and the retrieval of multiple arithmetic facts. Thus, children with low capacity in selective attention should suffer from more interference and confusion resulting from the manipulation of intermediate results. They should more easily loss track in the on going algorithm. Moreover, the difference between children with high and low capacity in selective attention should be amplified with the number of processing steps needed to solve a long division.

To summarize, performance should be a function of the number of processing steps and the spatial load induced by the long divisions, with a higher number of processing steps or higher spatial load resulting in lower performance. Consequently, children with higher attention capacity should outperform children with lower capacity, especially when long divisions require more processing steps. Similarly, children with better knowledge in multiplicative facts and better visuospatial skills should have better performance in solving long division.

Methods

Participants

Fifty six children (27 girls) in Grade 5 were recruited from four different classes in three French urban schools in socioeconomic middle class areas of the same city. They were aged between 10 years 2 months and 11 years 2 months (mean=10 years 8 months, SD=3.5 months) at the time of the study, which took place in the first semester of 2012. The ratio female/male was not significantly different between classes, $x^2 2<1$. None of the children presented diagnosed deficit. All children were French native speakers and they never had jumped or repeated a grade. All children received the permission of their caretakers to participate.

Material and procedure

Children did six different tests. Besides assessing children's performance in solving different types of long divisions, we measured for each child her knowledge of the multiplicative facts. We also evaluated individual differences in attentional capacity through two tests, the test d2 and the Cancellation subtest of the WISC, and differences in visuospatial planning capacity with the Mazes subtest and the Rover subtest from the KABC.

Long divisions

Twenty-one long divisions were chosen: 4 for Type 1, 5 for Type 2 and for Type 3, and 7 for Type 4 (Table 1). The divisions were presented on the right page of the booklet, leaving the left page blank that children could use as a draft. In the choice of the divisions and in their distribution into the four groups, several factors were controlled: Dividend, divisor and quotient did not include any zero, the intermediate subtractions did not involve any carry, and the intermediate multiplications did not involve any ties or five, the different multiplication tables being equally represented. The divisions were presented in a random order, the same for all children, which were instructed to solve them in this order, moving to the next one if they failed to solve one. Children had 7 min 30 sec for solving sets of 5 divisions and 9 min for the set of 6 divisions to keep an average time of 1 min 30 sec per division, which was sufficient for solving all the divisions as children finished before this limit. We analysed the percentage of correctly solved divisions, i.e., when both the quotient and remainder were correct.

Knowledge of multiplicative facts

The 80 multiplications that represent the operations in tables from 2 x 1 to 9 x 10 were presented on one page in a random order, identical for all children. The children had to solve them as fast as possible, proceeding in order column by column, going to the next multiplication if they did not know the answer. They had a maximum time of 2 minutes in accordance with the rate used in other studies with similar aged children [32,33]. We scored the number of multiplications correctly solved.

d2 test of attention

This test of selective and sustained attention consists of 14 lines of 47 characters [29]. These characters are the letters *p* and *d*, with one to four dashes above or underneath the letter. Children had to cross out all *d*s with two dashes (one above and one below, two above, or two below) as quickly as possible. According to the instructions of the test, a time limit of 20 seconds per line was given, after which a signal indicated to move to the next line. We computed the overall concentration index, which is the total number of targets correctly crossed out minus the number of errors, the maximum score being 299.

Cancellation Subtest of the WISCIV

In this subtest, the goal is to cross out all animals as quickly and accurately as possible among a set of different objects [34]. Children performed this task on two different sets, one without specific spatial organization, and one in which objects were in columns and lines. The time limit was 90 seconds for each set. We scored across the two sets the number of stimuli correctly crossed out, from which incorrectly crossed out stimuli were subtracted. Children could reach a maximum score of 128.

Mazes Subtest of the WISCIII

Children sought the shortest path to exit from mazes of increasing difficulty. They should not lift the pen from the paper and avoid entering into dead ends. In accordance with WISC instructions, the quotation was based on having or not found the exit, having or not jumped over a wall, and on the number of entries in dead ends. To shorten the procedure, we presented 6 mazes, which could lead to a maximum score of 19. Children had a maximum time of 9 min 15 sec for the 6 mazes, which was the sum of the duration allocated for each maze in WISC -III.

Rover Subtest of the KABC

The goal was to find the shortest path from a starting point to a bone on a draught board [28]. Unlike the original version in which children move a little dog figurine, children drew a circle on the boxes where the "dog" jumped. The child can move the dog in all directions, without passing on a bush, and rocks count double. Points are given for the optimal path or a path with one displacement more. To shorten the procedure, 6 trials were discarded from the original, leaving 13 trials. Children could obtain a maximum of 26 points. Two sample items were solved on the blackboard with the children. Instead of having a limited time for each trial, a total time of 10 min was given which is the sum of the time limits in the original version.

Each class was seen in a group session of about one and a half hour. The children were all given the same booklet with the six tasks in the following order: multiplications, d2 test, a first set of 5 long divisions, mazes subtest, a second set of 5 long divisions, cancellation subtest, a third set of 6 long divisions, rover subtest, and a fourth set of 5 long divisions. All paper and pencil tasks were completed following oral instructions given at the beginning of each task. Two adults supervised each class.

Results

The data from six children were discarded from the analysis because they reached a ceiling score at one of the subtests (one in Cancellation, four in Maze and one in Rover tests). Moreover, we did not introduce gender as a factor in the following analyses because female and males did not differ in the overall percentage of correctly solved divisions as well as in each type of divisions, ts<1. We first explored the characteristics of the operations that determine their difficulty. Second, we investigated cognitive abilities that may account for children's performance and their potential interaction with operation characteristics.

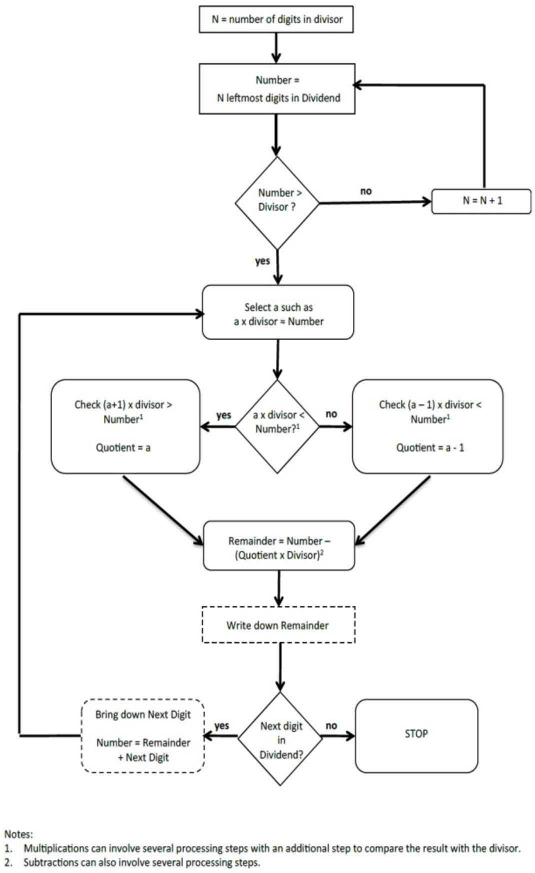

Figure 2: The algorithm for solving long divisions when children can rely on multiplication.

The characteristics of operations

The different types of divisions lead to variable performance. Types 3 and 4 (54%, SD=4, and 52%, SD=15, respectively) led to lower percentage of correct responses than Type 1 (80%, SD=8) and Type 2 (67%, SD=5) divisions (Table 1). This classification relies on the number of digits in the divisor and in the quotient. However, studies in the solving of other arithmetic operations including simple divisions [19] showed that the magnitude of operands is a determinant factor. This was not the case here. Due to the strong correlation between the number of digits and the magnitude of the four numbers involved in divisions (i.e., dividend, divisor, quotient and remainder), two separate stepwise regression analyses were performed on the percentage of correct responses with either their number of digits or their magnitude as predictors. In both analyses, the divisor and the dividend contributed unique variance, but the analysis with their magnitude led to a smaller R2 (0.47) than the analysis with their number of digits (R2=0.67). In this latter analysis, the number of digits of the divisor was a better predictor (R2=0.50, F(1, 19)=19.26, p<0.0001) than the number of digits of the dividend (R2 change =0.16, F(1, 18)=8.73, p=0.008).

We propose that the number of digits accounted for the performance in long divisions because it constrains the number of processing steps of the algorithm explicitly taught in classroom. However, within this algorithm, the number of processing steps depends on the strategy used to solve intermediate multiplications. Children can either compute multiplications (Figure 2 and Appendix for an example), perform recursive additions (Figure 3), or alternate between these two strategies depending on the difficulty of the problems. In this mixed strategy, children would use multiplication for the simplest divisions (i.e., with one digit divisors), but would back up to recursive additions for the most complex (i.e., with two digit divisors). Although the estimates resulting from these three alternative assumptions were all significantly correlated with performance in solving long divisions, the mixed strategy index was more strongly correlated (r=0.83, p<0.0001) than the indexes for the multiplication (r=0.64, p=0.002) and addition strategies (r=0.68, p=0.001). When the number of processing steps resulting from the mixed strategy was introduced in a stepwise regression analyses along with the number of digits in operands, it contributed unique variance, R2=0.68, F(1, 19)=40.39, p<0.0001. Only the number of digits in divisor added a significant contribution, R2 change=0.08, F (1, 18)=5.61, p=0.029.

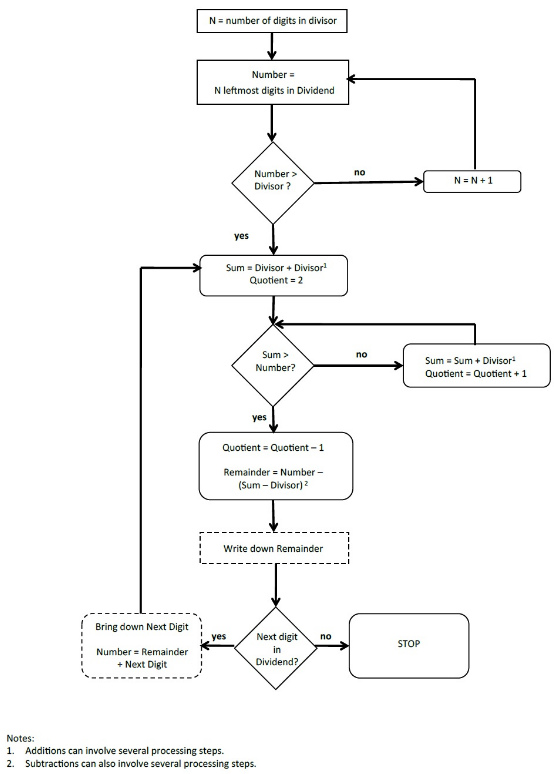

Figure 3: The algorithm for solving long divisions when children rely on recursive additions.

To complete these analyses, we computed for each division a spatial load corresponding to the number of digits that must be spatially arranged during its solving (Figure 1 and Table 1). It included the number of digits written down for the intermediate subtractions, the remainders and the digits of the dividend that were brought down to the remainders. Although this spatial load significantly correlated with the overall percentage of correctly solved divisions (r=-0.56, p=0.009), it did not explain any significant part of variance when introduced in a stepwise regression with the mixed strategy index.

Type	Problems				Number of Processing Steps			Spatial	Percent Correct
	Dividend	Divisor	Quotient	Remainder	Addition	Multiplication	Mixed	Load	
1	8	3	2	2	10	10	10	2	89
	7	4	1	3	8	10	10	2	82
	59	8	7	3	29	14	14	3	70
	56	9	6	2	27	14	14	3	
2	94	7	13	3	19	18	18	6	73
	556	6	92	4	41	22	22	8	70
	864	3	288	0	47	25	25	10	63
	4498	8	562	2	54	30	30	11	63
	7869	4	1967	1	65	34	34	14	68
3	99	26	3	21	18	15	18	4	52
	326	81	4	2	26	17	26	4	59
	366	63	5	51	29	19	29	5	57
	269	34	7	31	35	19	35	5	52
	699	72	9	51	44	19	44	5	50
4	768	36	21	12	22	25	22	8	63
	759	18	42	3	35	27	35	7	
	1489	62	24	1	37	29	37	10	55
	6589	43	153	10	47	36	47	16	54
	3898	81	48	10	61	28	61	11	46
	9699	61	159	0	68	33	68	15	55
	7993	27	296	1	82	40	82	15	21

Table 1: Percentage of correct responses in division solving with the number of processing steps and spatial load.

Children's cognitive abilities

Children's cognitive abilities were tested through five tests for which the descriptive statistics are provided in Table 2. The attention tests (i.e., d2 and cancellation tests) significantly correlated, r=0.28, p=0.05, whereas the correlation between the spatial tests (i.e., the mazes and rover tests) failed to reach significance, r=0.18, p=0.20, probably because the mazes test was not very discriminant and reliable as testified by the small SD and the low inter-trial correlation. We performed a Principal Component Analysis to extract an attention capacity factor from the d2 and Cancellation tests, and a spatial ability factor from the mazes and rover tests. A stepwise regression1 was performed using these factors along with knowledge of multiplicative facts as independent variables and percent correct on divisions as dependent variable. This analysis revealed two significant factors, knowledge of multiplicative facts, R2=0.15, F (1, 48)=8.54, p=0.005, and the attention capacity factor, R2=0.23, R2 change=0.08, F (1, 47)=4.80, p=0.03.

To test if performance in children with low attention capacity was more affected by increased number of processing steps, we contrasted three groups differing in attention capacity. As in individual difference studies by Engle and collaborators [35], the high attention capacity group included one third of the participants (n=17) with the higher capacity, the low attention capacity group included 17 participants with the lower attention. The remaining 16 participants were grouped under the medium capacity label. We discard the data from one division (7993/27), because its high number of processing steps and low performance strongly affected the regression slopes. As illustrated in Figure 4, the increase in the number of processing steps using the mixed strategy resulted in poorer performance in the three groups, the slopes of the regression line being significantly different from zero in high, medium and low capacity groups, t(16)=2.99, p=0.008, t(15)=4.45, p<0.0001, and t(16)=4.25, p<0.0001, respectively. Although the slope was smaller in high capacity individuals than in the two other groups, slopes did not significantly differ between groups as testified by the nonsignificant interaction between groups and number of processing steps in an ANCOVA with the number of processing steps as covariate, F<1. It should be noted that taking the entire sample into account (n=56), the interaction between groups and number of processing steps became significant, F(2, 54)=6.85, p=0.002. This effect was mainly due to the smaller slope exhibited by the high capacity group compared with the two other groups, F(1, 56)=11.23, p=0.001.

	Knowledge of facts facts	d2	Cancellation	Mazes	Rover
d2	**0.27**				
Cancellation	0.19	**0.28**			
Mazes	0.10	**0.28**	0.23	-	
Rover	0.19	**0.3**	0.19	0.18	
Division	0.39	**0.47**	0.14	0.16	0.09
Mean	36.50	113.20	91.00	14.30	17.28
SD	11.87	28.92	21.90	2.68	4.28
Obs. Min.	18	26	46	8	6
Obs. Max.	64	166	126	18	24
Reliability	0.967	0.913	0.608	0.09	0.647
Skewness	0.605	-0.736	-0.128	-0.574	-0.673
Kurtosis	0.029	1.123	-0.902	-0.594	0.063

Note: Significant correlations at p<0.05 in bold,. Obs. Min. and Obs. Max. are the observed minimum and maximum scores, respectively. Reliability calculated by correlating scores on half of the trials with the other half (odd vs. even trials).

Table 2: Correlation matrix and descriptive statistics for the five subtests.

Finally, and as we did for attention capacity, we contrasted three groups differing in their spatial abilities. The impact of spatial load on performance did not differ between children with high, medium, or low spatial abilities, as testified by the nonsignificant interaction between groups and spatial load, Fs<1 for both the restricted and the entire sample. This result confirmed the absence of significant effect of the spatial abilities in the regression analysis above Figure 4.

Discussion

To our knowledge, the current study is the first to explore the cognitive abilities contributing to children's performance in solving Euclidian or long divisions. It also aimed at characterising the factors that determine the difficulty of long divisions. Three main phenomena arose from the current results. First, the main predictor of problem difficulty is the number of processing steps needed to solve divisions. Second, children with better knowledge in multiplicative facts and higher attentional capacity outperform children with lower scores. Third, when considering our entire sample of participants, individual differences related to attentional capacities have a stronger effect on more difficult problems. These three issues are now discussed in turn.

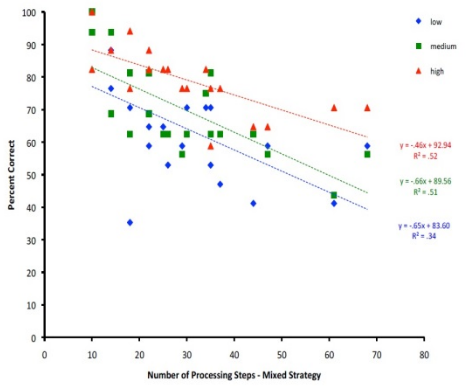

Figure 4: Mean percent correct for 20 operations (7993/27 being excluded) according to the number of processing steps needed to solve them with the mixed strategy for three groups of participants differing in their attention capacity (low, medium or high capacity).

The number of processing steps

Studies on arithmetic problem solving have shown that one of the main determinants of performance is the size of the operands, a phenomenon known as the problem size effect. Simple additions involving large numbers are more difficult than additions of smaller numbers, an effect that was replicated for subtractions, multiplications, and divisions [19]. However, the present study showed that the number of processing steps in the algorithm explicitly taught in French classrooms is a better predictor of the difficulty of divisions than the number of digits or the magnitude of the numbers involved in the problems. This suggests that the difficulty of long divisions relies on procedural demands, as we already observed for another numerical activity, the transcoding of numbers [36,37].

The ACT-R theory is especially appropriate to describe and formalize the cognitive processes in this algorithm [38]. The processing steps proposed in Appendix 1 are similar to procedural rules as defined in ACT-R. Procedural rules act on declarative knowledge to transform transient representations stored in working memory, and require sources of activation (or attention) for their implementation. This declarative knowledge is either encoded from the environment or retrieved from long term memory. For the long divisions, the processing steps of the algorithm act either on problem operands or on the multiplicative, subtractive and sometimes even additive facts retrieved from long term memory. Thus, performance in solving divisions relies on both the retrieval of previously stored declarative knowledge and the implementation of procedural rules. As a consequence, it should be expected that the level of proficiency in retrieving multiplicative facts and the amount of attention that could be dedicated to the implementation of processing steps are the two main cognitive abilities predicting children's performance in solving long divisions.

The knowledge of multiplicative facts and attentional capacity

Accordingly, our study revealed that individual differences in knowledge of multiplicative facts and in attentional capacity are the best predictors of 10-year-old children's performance in solving long divisions. It is not so surprising that when children have to solve long divisions, they rely on multiplicative facts. Although our study is the first to demonstrate this relationship for long divisions, we reported in the introduction several studies showing how important multiplicative facts are in solving simple divisions [17,19,39]. Even young adults, after years of practice, do not stop using multiplicative facts to solve divisions (e.g., converting 48/6 into 6 x ?=48). Having an efficient access to these facts would then favour the solving of long divisions, even more strongly than for simple divisions, because they require several retrievals of multiplicative facts. The importance for children to have a well established knowledge of multiplicative facts is even more apparent when considering that the main determinant of division difficulty was the number of processing steps for the mixed strategy, which assumes the use of recursive additions for solving intermediate multiplications involving two digit numbers.

The use of recursive additions to solve simple multiplications is the second mostly used strategy after retrieval in children. As a consequence, when children had to solve difficult divisions, they backed up to addition, an operation that benefits from both a stronger network of stored facts and a deeper practice of its algorithm as it is the first learned operation. The choice of using additions could result from two reasons. First, when divisions are more difficult, like Types 3 and 4, the number of processing steps increases, and consequently the amount of information to keep track. This imposes a supplementary cognitive load that reduces the attention available for solving intermediate multiplications. Under such cognitive load, children could favour a less demanding strategy. The second possible reason is that difficult divisions involve larger numbers and, consequently, complex multiplications (e.g., 43 x 5 in Figure 1). Because the algorithm for complex multiplication is a relatively new skill for 10-year-old children, they may favour the more secure and well practiced algorithm for addition. These two reasons are not exclusive from each other.

Apart from knowledge of multiplicative facts, performance in solving long divisions was also affected by attentional capacity. The current study brought thus a further example of the impact on cognitive performance of individual differences in selective attention. Such finding could also be reminiscent to the impact of another type of attention, namely executive or controlled attention, in mathematical cognition [39,40-45]. Solving long divisions relies on a multistep process requiring the retrieval of information in long term memory. Individual differences in selective attention could affect the efficiency of each processing step, of the retrieval process as well as the storage of intermediary results. As shown in adults, individuals with higher attentional capacity are more resistant to interference [35], which should facilitate retrieval of multiplicative or additive facts among a highly interfering network [46]. As a consequence, it can be expected that the difference between high and low attentional capacity children increases with the number of processing steps to implement and the number of needed retrievals. In other words, the two groups should largely differ in solving the more complex divisions.

The present findings lent some support to this expectation. When considering the entire sample of participants, the increase in the number of processing steps amplified the difference in performance between high attentional capacity children and their peers, the latter being more affected than the former by this increase. This finding is similar to what we have already observed in transcoding numbers, for which the rate of transcoding errors increased when more production rules were required, an effect that was more pronounced for children with low than high controlled attention capacities [37]. This phenomenon of individual differences increasing with task complexity can be accounted for by the hypothesis of a concatenation of small differences in every elementary component of the task, as demonstrated by Barrouillet et al. [37]. Because individuals with higher attentional capacities outperform their peers in elementary activities such as reading digits, solving simple problems, retrieving items of knowledge from longterm memory and implementing atomic procedural rules, their higher efficiency becomes more and more evident as the complexity of the task increases, as it is the case with long divisions that necessitate all these elementary components.

Conclusion

One year after the start of formal instruction, children still experience difficulties in solving long divisions. Beyond the learning of a complex and multistep algorithm, children's performance is dependent on the implementation of each processing step. These processing steps require the retrieval of multiplicative, subtractive, and additive facts, and the use of algorithms for multiplication, subtraction and addition. As a consequence, to solve a long division, a child has to draw on all the arithmetic knowledge acquired in primary school, and to coordinate this various knowledge according to a newly learned algorithm. The fact that long division is a synthesis of all arithmetic knowledge explains why long division is considered as a major source of difficulty in children, and one of the biggest challenges for instruction in primary school.

This study has thus important implication for teaching in primary school. As we have seen, solving long divisions is made more difficult by the fact that children have to use recursive additions instead of retrieving the multiplicative facts. This result emphasizes the need for a stronger practice in the learning of multiplicative facts. Helping children to acquire a secure knowledge of the multiplicative facts, that is to allow them to easily retrieve these facts directly from long term memory, it would reduce the use of the recursive additions, diminishing the number of processing steps, which is the major determinant for children's performance in solving long divisions.

Finally, this study opens a new field of research about an arithmetic operation, division, which is poorly studied. We would like to suggest two future lines of research that would nicely extend the current study. First, we have seen that visuospatial planning does not seem to play an important role in solving long divisions. However, the two tests used in the present study were poorly correlated to each other. As a consequence, it might be interesting to assess the individual differences in visuospatial planning with other tests. Alternatively, it could also be suggested that the role of visuospatial planning in solving long divisions is stronger when children are learning the procedure of this operation, and at 10 years of age with some mastery of the procedure the organisation of digits is not a source of difficulty. A second line of research could focus on understanding how children learn to coordinate the different knowledge required to solve long divisions. As mentioned above, this coordination could explain why division among the other operations remains the most difficult.

Footnote

Introducing the five subtests in the analysis lead to similar results with only the d2 test, $R2=0.22$, $F(1, 48)=13.44$, $p=0.001$, and the knowledge of multiplicative facts, $R2=0.29$, $R2$ change=0.07, $F(1, 47)=4.50$, $p=0.03$, contributing unique variance.

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Multiplication strategy		Addition strategy	
# Steps	Action	# Steps	Action
1	N = 2	1	N = 2
2	Number = 36	2	Number = 36
3	36 > 63? no	3	36 > 63? no
4	N = 2+1= 3	4	N = 2+1= 3
5	Number = 366	5	Number = 366
6	366 > 63? yes	6	366 > 63? yes
7	a = 6	7-9	63 + 63 = 126 126 > 366 ? no
8-10	6 x 63 = 378	10-12	126 + 63 = 189 189 > 366? no
11	378 < 366 ? no	13-17	189 + 63 = 252 252 > 366? no
12-14	5 x 63 = 315	18-21	252 + 63 = 315 315 > 366? no
15	Check 315 > 366 Quotient = 5	22-24	315 + 63 = 378 378 > 366? yes
16-18	Remainder = 366 - 315 = 51	25	Quotient = 5
	Write down 51	26-28	Remainder = 366 – 315 = 51
19	Next digit? no		Write down 51
	STOP	29	Next digit? no
			STOP
19 processing steps		29 processing steps	

Appendix: An example of the algorithm of the division 366/63 with the multiplicative and the additive strategy.

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